Statik

Reaktion, moment, nedbøjning m.m. for bjælker med forskellig understøtning og belastning
af civilingeniør IDA Torben Klausen, Byggeteknisk Højskole, København

Til brug for overslagsdimensionering af bjælker findes reaktioner, momenter og udbøjninger for bjælker med forskellige understøtninger og laster på denne og de efterfølgende sider.

Overslagsdimensioneringen, der altid senere bør verificeres af en statiker, bør både omfatte styrke og stivhed, idet udbøjningskrav i dag ofte vil være dimensionsgivende.

Permanent last er typisk egenlast. Variabel last er typisk nyttelast, snelast og/eller vindlast.

Styrke

Bjælken udsættes for en regningsmæssig last, der bestemmes af,

ved fordelt last:
$q = 1{,}0 \cdot (\text{permanent last}) + 1{,}3 \cdot (\text{variabel last})$.

ved punktlast:
$Q = 1{,}0 \cdot (\text{permanent last}) + 1{,}3 \cdot (\text{variabel last})$.

Det eftervises, at $M_d/W \le f_d$.

Stivhed

Bjælken udsættes for en regningsmæssig last, der bestemmes af,

- ved fordelt last:
$q = 1{,}0 \cdot (\text{permanent last}) + 0{,}5 \cdot (\text{variabel last})$.

- *ved punktlast:*
$Q = 1{,}0 \cdot (\text{permanent last}) + 0{,}5 \cdot (\text{variabel last})$.

Det eftervises, at $u_d \le L/300$.

Betegnelser

A, B, C, D: Punkter.

a: Længde (mm).

E: Elasticitetsmodul (N/mm^2). Til overslag kan anvendes:

Stål	Træ L40/Kerto	Træ L30/K30	Træ K24	Træ K18
210.000	8400	7200	6300	5400

f_d: Regningsmæssig bøjningsstyrke (N/mm^2). Til overslag kan anvendes:

Stål	Træ L40	Træ L30/K30	Træ K24	Træ K18
184	17,8	13,0	11,2	9,0

J: Inertimoment (mm^4).

L: Længde (mm).

M_A: Moment i punkt A (N·mm).

M_B: Moment i punkt B (N·mm).

M_{AB}: Maximalt/minimalt moment mellem punkt A og punkt B (N·mm).

M_d: Dimensionsgivende moment (N·mm).

M_x: Moment i afstanden x fra punkt A (N·mm).

Q: Punktlast (N).

q: Linielast, dvs. fordelt last (N/mm).

R_A: Reaktion i punkt A (N).

R_B: Reaktion i punkt B (N).

u_{AB}: Maximal udbøjning mellem punkt A og punkt B (mm).

u_B: Udbøjning i punkt B (mm).

u_C: Udbøjning i punkt C (mm).

u_D: Udbøjning i punkt D (mm).

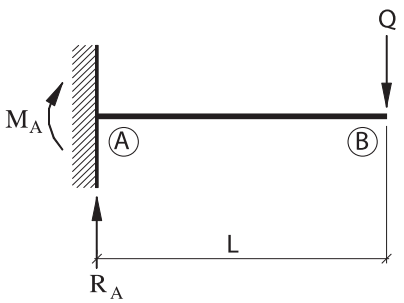
u_d: Dimensionsgivende udbøjning (mm).

u_x: Udbøjning i afstanden x fra punkt A (mm).

W: Modstandsmoment (mm^3).

x: Absisse, afstand fra punkt A (mm).

1.

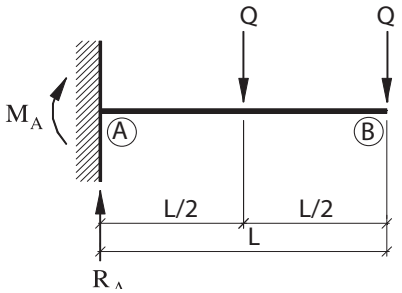


$$R_A = Q$$

$$M_d = -M_A = Q \cdot L$$

$$u_d = u_B = \frac{1}{3} \cdot \frac{Q \cdot L^3}{E \cdot J}$$

2.



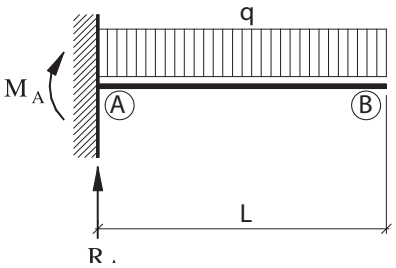
$$R_A = 2 \cdot Q$$

$$M_d = -M_A = \frac{3}{2} \cdot Q \cdot L$$

$$u_d = u_B = \frac{7}{16} \cdot \frac{Q \cdot L^3}{E \cdot J}$$

Når bjælken belastes med N punktlaster af størrelsen Q, én Q-last i punkt B og resterende Q-laster fordelt i bjælkens N'te-delspunkter, kan M_A og u_B beregnes som for en bjælke med jævnt fordelt linielast q, hvor q = (N + 1) · Q / L. Afvigelse på M_A er 0% og afvigelse på u_B er mindre end 10%, når N ≥ 3.

3.

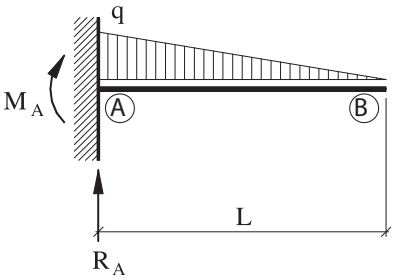


$$R_A = q \cdot L$$

$$M_d = -M_A = \frac{1}{2} \cdot q \cdot L^2$$

$$u_d = u_B = \frac{1}{8} \cdot \frac{q \cdot L^4}{E \cdot J}$$

4.

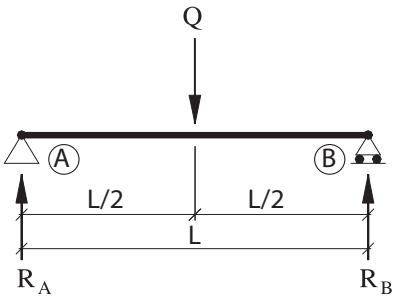


$$R_A = \frac{1}{2} \cdot q \cdot L$$

$$M_d = -M_A = \frac{1}{6} \cdot q \cdot L^2$$

$$u_d = u_B = \frac{1}{30} \cdot \frac{q \cdot L^4}{E \cdot J}$$

5.

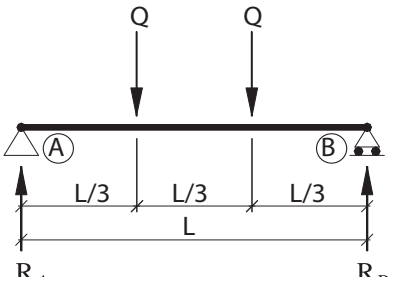


$$R_A = R_B = \frac{1}{2} \cdot Q$$

$$M_d = M_{AB} = \frac{1}{4} \cdot Q \cdot L \quad \text{for} \quad x = \frac{1}{2} \cdot L$$

$$u_d = u_{AB} = \frac{1}{48} \cdot \frac{Q \cdot L^3}{E \cdot J} \quad \text{for} \quad x = \frac{1}{2} \cdot L$$

6.



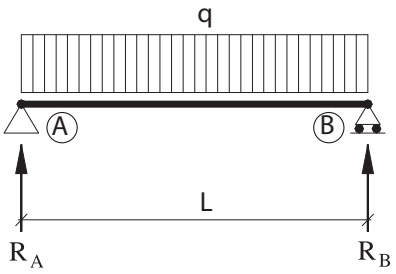
$$R_A = R_B = Q$$

$$M_d = M_{AB} = \frac{1}{3} \cdot Q \cdot L \quad \text{for} \quad \frac{1}{3} \cdot L \le x \le \frac{2}{3} \cdot L$$

$$u_d = u_{AB} = \frac{23}{648} \cdot \frac{Q \cdot L^3}{E \cdot J} \quad \text{for} \quad x = \frac{1}{2} \cdot L$$

Når bjælken belastes med N punktlaster af størrelsen Q og fordelt i bjælkens (N+1)'te-delspunkter, kan M_{AB} og u_{AB} beregnes som for en bjælke med jævnt fordelt linielast q, hvor q=(N+1)·Q/L. Afvigelsen på M_{AB} og på u_{AB} er mindre end 5%, når N≥3.

7.

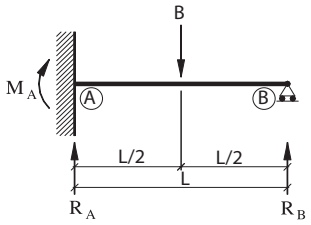


$$R_A = R_B = \frac{1}{2} \cdot q \cdot L$$

$$M_d = M_{AB} = \frac{1}{8} \cdot q \cdot L^2 \quad \text{for} \quad x = \frac{1}{2} \cdot L$$

$$u_d = u_{AB} = \frac{5}{384} \cdot \frac{q \cdot L^4}{E \cdot J} \quad \text{for} \quad x = \frac{1}{2} \cdot L$$

8.

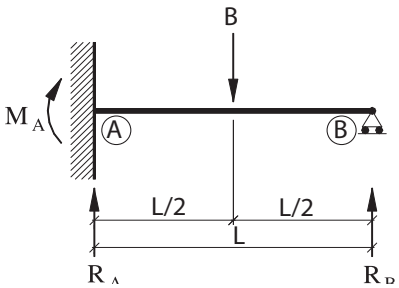


$$R_A = \frac{11}{16} \cdot Q \qquad R_B = \frac{5}{16} \cdot Q$$

$$M_d = -M_A = \frac{3}{16} \cdot Q \cdot L \qquad M_{AB} = \frac{5}{32} \cdot Q \cdot L \quad \text{for} \quad x = \frac{1}{2} \cdot L$$

$$u_d = u_{AB} = \frac{\sqrt{5}}{240} \cdot \frac{Q \cdot L^3}{E \cdot J} \quad \text{for} \quad x = \left(1 - \frac{\sqrt{5}}{5}\right) \cdot L$$

9.

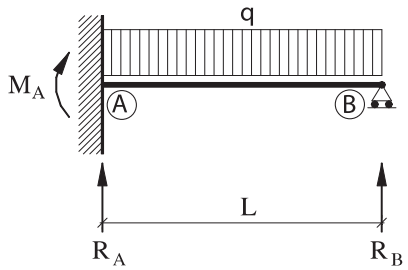


$$R_A = \frac{11}{16} \cdot Q \qquad R_B = \frac{5}{16} \cdot Q$$

$$M_d = -M_A = \frac{3}{16} \cdot Q \cdot L \qquad M_{AB} = \frac{5}{32} \cdot Q \cdot L \quad \text{for} \quad x = \frac{1}{2} \cdot L$$

$$u_d = u_{AB} = \frac{\sqrt{5}}{240} \cdot \frac{Q \cdot L^3}{E \cdot J} \quad \text{for} \quad x = \left(1 - \frac{\sqrt{5}}{5}\right) \cdot L$$

10.

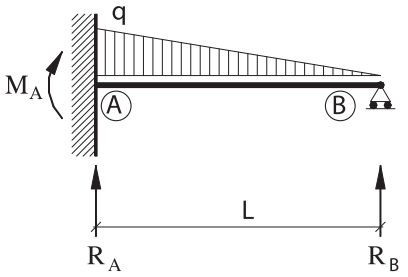


$$R_A = \frac{5}{8}\cdot q\cdot L \qquad R_B = \frac{3}{8}\cdot q\cdot L$$

$$M_d = -M_A = \frac{1}{8}\cdot q\cdot L^2 \qquad M_{AB} = \frac{9}{128}\cdot q\cdot L^2 \quad \text{for} \quad x = \frac{5}{8}\cdot L$$

$$u_d = u_{AB} = \frac{1}{185}\cdot\frac{q\cdot L^4}{E\cdot J} \quad \text{for} \quad x = \frac{15-\sqrt{33}}{16}\cdot L$$

11.



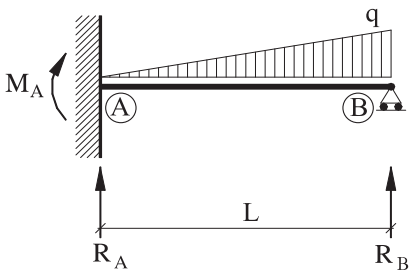
$$R_A = \frac{2}{5}\cdot q\cdot L \qquad R_B = \frac{1}{10}\cdot q\cdot L$$

$$M_d = -M_A = \frac{1}{15}\cdot q\cdot L^2$$

$$M_{AB} = \frac{\sqrt{5}}{75}\cdot q\cdot L^2 \quad \text{for} \quad x = \left(1-\frac{\sqrt{5}}{5}\right)\cdot L$$

$$u_d = u_{AB} = \frac{2\cdot\sqrt{5}}{1875}\cdot\frac{q\cdot L^4}{E\cdot J} \quad \text{for} \quad x = \left(1-\frac{\sqrt{5}}{5}\right)\cdot L$$

12.



$$R_A = \frac{9}{40}\cdot q\cdot L \qquad R_B = \frac{11}{40}\cdot q\cdot L$$

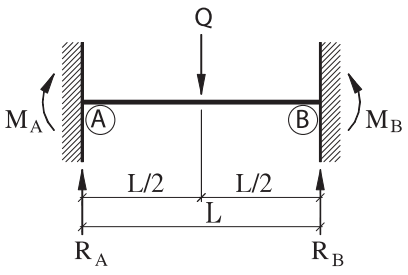
$$M_d = -M_A = \frac{7}{120}\cdot q\cdot L^2$$

$$M_{AB} = \frac{81\cdot\sqrt{5}-105}{1800}\cdot q\cdot L^2 \quad \text{for} \quad x = \frac{3\cdot\sqrt{5}}{10}\cdot L$$

Udbøjningen bestemmes ved iteration

$$u_d = u_{AB} = 0{,}00305\cdot\frac{q\cdot L^4}{E\cdot J} \quad \text{for} \quad x = 0{,}598\cdot L$$

13.

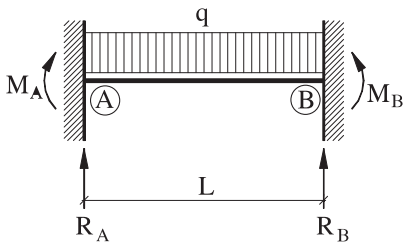


$$R_A = R_B = \frac{1}{2}\cdot Q$$

$$M_d = -M_A = -M_B = M_{AB} = \frac{1}{8}\cdot Q\cdot L \quad M_{AB} \text{ for } x = \frac{1}{2}\cdot L$$

$$u_d = u_{AB} = \frac{1}{192}\cdot\frac{Q\cdot L^3}{E\cdot J} \quad \text{for} \quad x = \frac{1}{2}\cdot L$$

14.

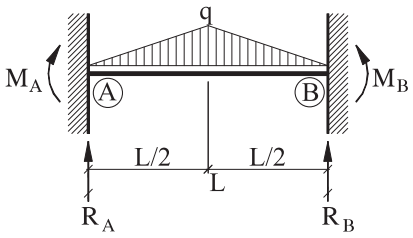


$$R_A = R_B = \frac{1}{2}\cdot q\cdot L$$

$$M_d = -M_A = -M_B = \frac{1}{12}\cdot q\cdot L^2 \quad M_{AB} = \frac{1}{24}\cdot q\cdot L^2 \text{ for } x = \cdot$$

$$u_d = u_{AB} = \frac{1}{384}\cdot\frac{q\cdot L^4}{E\cdot J} \quad \text{for} \quad x = \frac{1}{2}\cdot L$$

15.

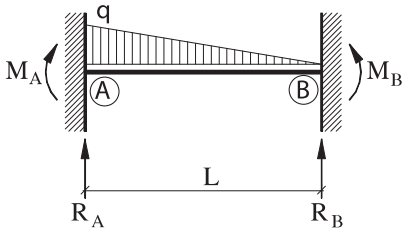


$$R_A = R_B = \frac{1}{4}\cdot q\cdot L$$

$$M_d = -M_A = -M_B = \frac{5}{96}\cdot q\cdot L^2 \quad M_{AB} = \frac{1}{32}\cdot q\cdot L^2 \text{ for } x = \frac{1}{2}\cdot L$$

$$u_d = u_{AB} = \frac{1}{549}\cdot\frac{q\cdot L^4}{E\cdot J} \quad \text{for} \quad x = \frac{1}{2}\cdot L$$

16.



$$R_A = \frac{7}{20}\cdot q\cdot L \qquad R_B = \frac{3}{20}\cdot q\cdot L$$

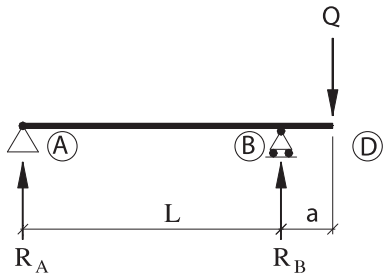
$$M_d = -M_A = \frac{1}{20}\cdot q\cdot L^2 \qquad M_B = -\frac{1}{30}\cdot q\cdot L^2$$

$$M_{AB} = \frac{\sqrt{270}-10}{300}\cdot q\cdot L^2 \quad \text{for} \quad x = \left(1-\sqrt{\frac{3}{10}}\right)\cdot L$$

Udbøjningen bestemmes ved iteration

$$u_d = u_{AB} = 0{,}00131\cdot\frac{q\cdot L^4}{E\cdot J} \quad \text{for} \quad x = 0{,}475\cdot L$$

17.



$$R_A = -Q\cdot\frac{a}{L} \qquad R_B = Q\cdot\left(1+\frac{a}{L}\right)$$

$$M_d = -M_B = Q\cdot a$$

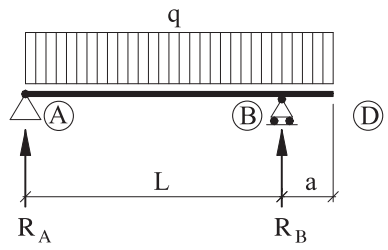
$$u_d = -u_{AB} = \frac{\sqrt{3}}{27}\cdot\frac{Q\cdot L^3}{E\cdot J}\cdot\left(\frac{a}{L}\right) \quad \text{for} \quad x = \frac{\sqrt{3}}{3}\cdot L$$

eller

$$u_d = u_D = \frac{1}{3}\cdot\frac{Q\cdot L^3}{E\cdot J}\cdot\left(\left(\frac{a}{L}\right)^2+\left(\frac{a}{L}\right)^3\right)$$

OPSLAG

18.



$$R_A = \frac{1}{2}\cdot q\cdot L\cdot\left(1-\left(\frac{a}{L}\right)^2\right) \qquad R_B = \frac{1}{2}\cdot q\cdot L\cdot\left(1+\left(\frac{a}{L}\right)\right)^2$$

$$M_d = M_{AB} = \frac{1}{8}\cdot q\cdot L^2\cdot\left(1-\left(\frac{a}{L}\right)^2\right)^2 \quad \text{for} \quad x = \frac{1}{2}\cdot L\cdot\left(1-\left(\frac{a}{L}\right)^2\right)$$

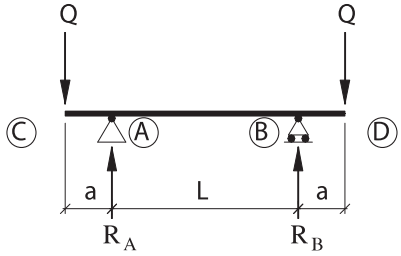
eller $M_d = -M_B = \frac{1}{2}\cdot q\cdot a^2$

$$u_d = u_D = \frac{1}{24}\cdot\frac{q\cdot L^4}{E\cdot J}\cdot\left(-\left(\frac{a}{L}\right)+4\cdot\left(\frac{a}{L}\right)^3+3\cdot\left(\frac{a}{L}\right)^4\right)$$

eller for $0 \le x \le L$

$$u_d = u_x = \frac{1}{24}\cdot\frac{q\cdot L^4}{E\cdot J}\cdot\left(\left(1-2\cdot\left(\frac{a}{L}\right)^2\right)\cdot\left(\frac{x}{L}\right)-2\cdot\left(1-\left(\frac{a}{L}\right)^2\right)\cdot\left(\frac{x}{L}\right)^3+\left(\frac{x}{L}\right)^4\right)$$

19.



$$R_A = R_B = Q$$

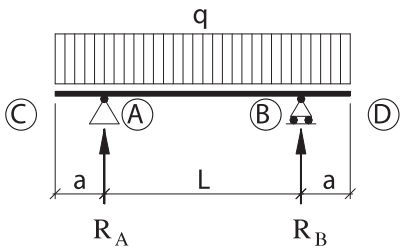
$$M_d = -M_A = -M_B = -M_{AB} = Q\cdot a$$

$$u_d = -u_{AB} = \frac{1}{8}\cdot\frac{Q\cdot L^3}{E\cdot J}\cdot\left(\frac{a}{L}\right) \quad \text{for} \quad x = \frac{1}{2}\cdot L$$

eller

$$u_d = u_C = u_D = \frac{1}{6}\cdot\frac{Q\cdot L^3}{E\cdot J}\cdot\left(3\cdot\left(\frac{a}{L}\right)^2+2\cdot\left(\frac{a}{L}\right)^3\right)$$

20.



$$R_A = R_B = \frac{1}{2}\cdot q\cdot L\cdot\left(1+2\cdot\left(\frac{a}{L}\right)\right)$$

$$M_d = M_{AB} = \frac{1}{8}\cdot q\cdot L^2\cdot\left(1-4\cdot\left(\frac{a}{L}\right)^2\right) \quad \text{for} \quad x = \frac{1}{2}\cdot L$$

eller $M_d = -M_A = -M_B = \frac{1}{2}\cdot q\cdot a^2$

$$u_d = u_C = u_D = \frac{1}{24}\cdot\frac{q\cdot L^4}{E\cdot J}\cdot\left(-\left(\frac{a}{L}\right)+6\cdot\left(\frac{a}{L}\right)^3+3\cdot\left(\frac{a}{L}\right)^4\right)$$

eller for $0 \le x \le L$

$$u_d = u_x = \frac{1}{24}\cdot\frac{q\cdot L^4}{E\cdot J}\cdot\left(\left(1-6\cdot\left(\frac{a}{L}\right)^2\right)\cdot\left(\frac{x}{L}\right)+6\cdot\left(\frac{a}{L}\right)^2\cdot\left(\frac{x}{L}\right)^2-2\cdot\left(\frac{x}{L}\right)^3+\left(\frac{x}{L}\right)^4\right)$$