

Assignment 2

Problem 1:

Question 1.1:

In `ambiguous(+word)` the goal is to check if there is more than one category. This is done by checking (for the same word) if the next wordclass is different from the previous.

Some tests:

```
?- ambiguous(above).
```

```
true.
```

```
?- ambiguous(abroad).
```

```
false.
```

Question 1.2

`display(n)` is showing the `n` most frequent words (with the `WordClass`).

It runs recursively and is comparing the `SortOrder` to `n` and if `n` is the largest of the two then it makes the printout.

Here is a test run:

```
?- display(500).
```

```
a det
```

```
able a
```

```
about adv
```

```
about prep
```

```
true
```

Problem 2

Question 2.1

`firstlast(?List)` runs recursively down to a list of two elements that are the same (the first and last element).

In between it cuts off the tail one by one.

Here are some tests:

```
?- firstlast([1,2,3,4,1]).
```

true.

```
?- firstlast([1,1]).
```

true.

```
?- firstlast([1,2,3,4,5]).
```

false.

```
?- firstlast([1]).
```

false.

```
?- firstlast([], []).
```

true.

Question 2.2

`fistlasta(?List)` is using `append` and merges the first lists head (we don't care about the tail) with a list of the same element to the input List. If it is the same then it returns true.

Here are some tests:

```
?- fistlasta([1,2,1]).
```

true.

```
?- fistlasta([1,2,3]).
```

false.

```
?- fistlasta([1]).
```

false.

```
?- fistlasta([], []).
```

true.

Problem 3

Question 3.1

Q 3.1 $(p \wedge q) \rightarrow (q \wedge p)$

Semantic tableau:

\neg $\neg((p \wedge q) \rightarrow (q \wedge p))$

α $(p \wedge q), \neg(q \wedge p)$

α $p, q, \underline{\neg(q \wedge p)}$

β $p, q, \neg q$ $p, q, \neg p$

\times \times

Valid

Question 3.2

CNF
 Resolution Procedure:

$A \rightarrow B \equiv \neg A \vee B$: $\neg((P \wedge Q) \rightarrow (Q \wedge P))$

De Morgan : $\neg(\neg(P \wedge Q) \vee (Q \wedge P))$

$\neg\neg A \equiv A$: $\neg\neg(P \wedge Q) \wedge \neg(Q \wedge P)$

De Morgan : $(P \wedge Q) \wedge \neg(Q \wedge P)$

$P \wedge Q \wedge (\neg Q \vee \neg P)$

$\{P\}, \{Q\}, \{\neg Q, \neg P\}$

1. P
2. Q
3. $\neg Q, \neg P$
4. $\neg Q$ 1,3
5. \square 4,2

Valid

Question 3.3

Q3.3
 Gentzen System of the formula:

1. $\vdash \neg P, \neg Q, Q$ Axiom
2. $\vdash \neg P, \neg Q, P$ Axiom
3. $\vdash \neg P, Q, (Q \wedge P)$ $\beta \wedge, 1, 2$
4. $\vdash \neg(P \wedge Q), (Q \wedge P)$ $\alpha \wedge, 3$
5. $\vdash (P \wedge Q) \rightarrow (Q \wedge P)$ $\alpha \rightarrow, 4$

Question 3.4

Q 3.4

Hilbert system \mathcal{H} of the formula $(P \wedge Q) \rightarrow (Q \wedge P)$:

- | | | |
|--|--|--|
| <p>Rule 3.26
 $\cup \vdash A \rightarrow B$
 $\cup \vdash \neg B \rightarrow \neg A$</p> | <p>1. $\{Q \rightarrow \neg P, \neg \neg P\} \vdash Q \rightarrow \neg P$
 2. $\{Q \rightarrow \neg P, \neg \neg P\} \vdash (Q \rightarrow \neg P) \rightarrow (\neg \neg P \rightarrow \neg Q)$
 3. $\{Q \rightarrow \neg P, \neg \neg P\} \vdash \neg \neg P \rightarrow \neg Q$
 4. $\{Q \rightarrow \neg P, \neg \neg P\} \vdash \neg \neg P$
 5. $\{Q \rightarrow \neg P, \neg \neg P\} \vdash \neg Q$
 6. $\{Q \rightarrow \neg P, \neg \neg P\} \vdash \neg Q \rightarrow \neg \neg \neg Q$
 7. $\{Q \rightarrow \neg P, \neg \neg P\} \vdash \neg \neg \neg P$
 8. $\{Q \rightarrow \neg P\} \vdash \neg \neg P \rightarrow \neg \neg \neg Q$
 9. $\{Q \rightarrow \neg P\} \vdash \neg \neg Q \rightarrow \neg P$
 10. $\{Q \rightarrow \neg P\} \vdash P \rightarrow \neg Q$
 11. $\vdash (Q \rightarrow \neg P) \rightarrow (P \rightarrow \neg Q)$
 12. $\vdash ((Q \rightarrow \neg P) \rightarrow (P \rightarrow \neg Q)) \rightarrow (\neg(P \rightarrow \neg Q) \rightarrow \neg(Q \rightarrow \neg P))$
 13. $\vdash \neg(P \rightarrow \neg Q) \rightarrow \neg(Q \rightarrow \neg P)$
 14. $\vdash (P \wedge Q) \rightarrow (Q \wedge P)$</p> | <p>Assumption
 Theorem 3.25
 $[(A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A)]$
 MP 1,2
 Assumption
 MP 3,4
 $[A \rightarrow \neg \neg A]$
 Theorem 3.23
 MP 5,6
 Deduction 7
 Contrapositive 8 Rule 3.15
 Contrapositive 9 Rule 3.15
 Deduction 10 Theorem 3.14
 Theorem 3.25
 $[(A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A)]$
 MP 11,12
 page 55.
 Def. of \wedge
 page 62.</p> |
|--|--|--|

Rule 3.24 (Double negation Rule) can be used on step 8 to 10 to get rid of the negation signs.

Rule 3.26 (the other direction of Contrapositive rule) can be used on step 1 to 3 instead. Also step 11 to 13.