Simon Benfeldt Jørgensen - s103182
DTU course 02156 Logical Systems and Logic Programming

Assignment 3

Problem 1

Q 1.1

From the code given the following query is explained:

```
?- tt(p imp q,[(p,t),(q,f)],f).

Yes
```

The query means that the truth table values for $p \rightarrow q$ is false when p is true and q is false. The reason is that the code finds the value of p (TVA) which is true and then the value of q (TVB) which is false and after that runs `opr(imp, t, f, TV)` where TV is the unknown value supposed to be found.

Q 1.2

The program `boolean` checks if the predicates `negate` and `opr` succeeds correctly corresponding to two-valued propositional logic.

This is done by looking at all the the 4 cases ([t,t], [t,f], [f,t], [f,f]) for the different operators and checking if the value is correct. If so the program succeeds otherwise it fails.

After the run:

```
?- boolean.

true.
```

Q 1.3

The opr with 4 inputs is made out of the rules from assignment 1 for many-valued logic. Here is the run for opr:

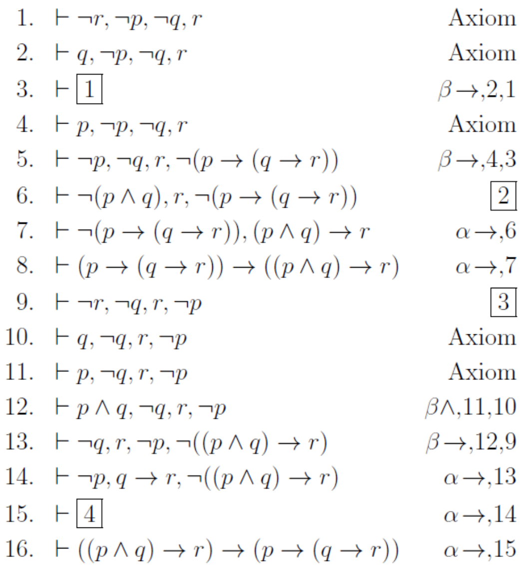
```
1 ?- opr.
    con eqv dis imp
t t  t   t   t   t
t f  f   f   t   f
t x  x   x   t   x
f t  f   f   t   t
f f  f   f   f   t
f x  f   f   x   t
x t  x   x   t   t
x f  f   f   x   x
x x  x   x   x   t
true.
```

Q 1.4

The boolean program succeeds because the truth table values for boolean logic is the same in many valued logic.

Problem 2

The Gentzen rules are used for the following:



1.	$\vdash \neg r, \neg p, \neg q, r$	Axiom
2.	$\vdash q, \neg p, \neg q, r$	Axiom
3.	$\vdash \boxed{1}$	$\beta\rightarrow$,2,1
4.	$\vdash p, \neg p, \neg q, r$	Axiom
5.	$\vdash \neg p, \neg q, r, \neg(p \rightarrow (q \rightarrow r))$	$\beta\rightarrow$,4,3
6.	$\vdash \neg(p \wedge q), r, \neg(p \rightarrow (q \rightarrow r))$	$\boxed{2}$
7.	$\vdash \neg(p \rightarrow (q \rightarrow r)), (p \wedge q) \rightarrow r$	$\alpha\rightarrow$,6
8.	$\vdash (p \rightarrow (q \rightarrow r)) \rightarrow ((p \wedge q) \rightarrow r)$	$\alpha\rightarrow$,7
9.	$\vdash \neg r, \neg q, r, \neg p$	$\boxed{3}$
10.	$\vdash q, \neg q, r, \neg p$	Axiom
11.	$\vdash p, \neg q, r, \neg p$	Axiom
12.	$\vdash p \wedge q, \neg q, r, \neg p$	$\beta\wedge$,11,10
13.	$\vdash \neg q, r, \neg p, \neg((p \wedge q) \rightarrow r)$	$\beta\rightarrow$,12,9
14.	$\vdash \neg p, q \rightarrow r, \neg((p \wedge q) \rightarrow r)$	$\alpha\rightarrow$,13
15.	$\vdash \boxed{4}$	$\alpha\rightarrow$,14
16.	$\vdash ((p \wedge q) \rightarrow r) \rightarrow (p \rightarrow (q \rightarrow r))$	$\alpha\rightarrow$,15

Q 2.1

The Beta rule for implication is used where β1 is q and β2 is ¬(¬r) which is just r.

[1]: *¬(q →r)*

The alpha rule for line 5 is used where ¬p and ¬q becomes ¬(*p ^ q)*

[2]: *α ^, 5*

Q 2.2

[3]: *Axiom*

The implication alpha rule is used:

[4]: *(p → (q → r)), ¬((p ^ q) → r)*

Q 2.3

To get back to the original formula, the biconditional is used. Since line 16 and 8 share respectively the left and right side of the implication we get:

[17]: ⊦ *((p ^ q) → r) ↔ (p → (q → r))* *β* ↔,16,8

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Problem 3

Q 3.1

Tableau for $(\exists x \forall y p(x,y)) \rightarrow (\forall y \exists x p(x,y))$:

$$\neg\big((\exists x \forall y p(x,y)) \rightarrow (\forall y \exists x p(x,y))\big)$$

$$\downarrow$$

$$\boldsymbol{\big((\exists x \forall y p(y,x))\big)},\ \neg(\forall y \exists x p(x,y))$$

$$\downarrow$$

$$\forall x p(a,x),\ \neg\boldsymbol{(\forall y \exists x p(x,y))}$$

$$\downarrow$$

$$\forall x p(a,x),\ \neg\boldsymbol{(\exists x p(x,b))}$$

$$\downarrow$$

$$\boldsymbol{\forall x p(a,x)},\ \neg(\exists x p(x,b)),\ \neg p(a,b),\ \neg p(b,b)$$

$$\downarrow$$

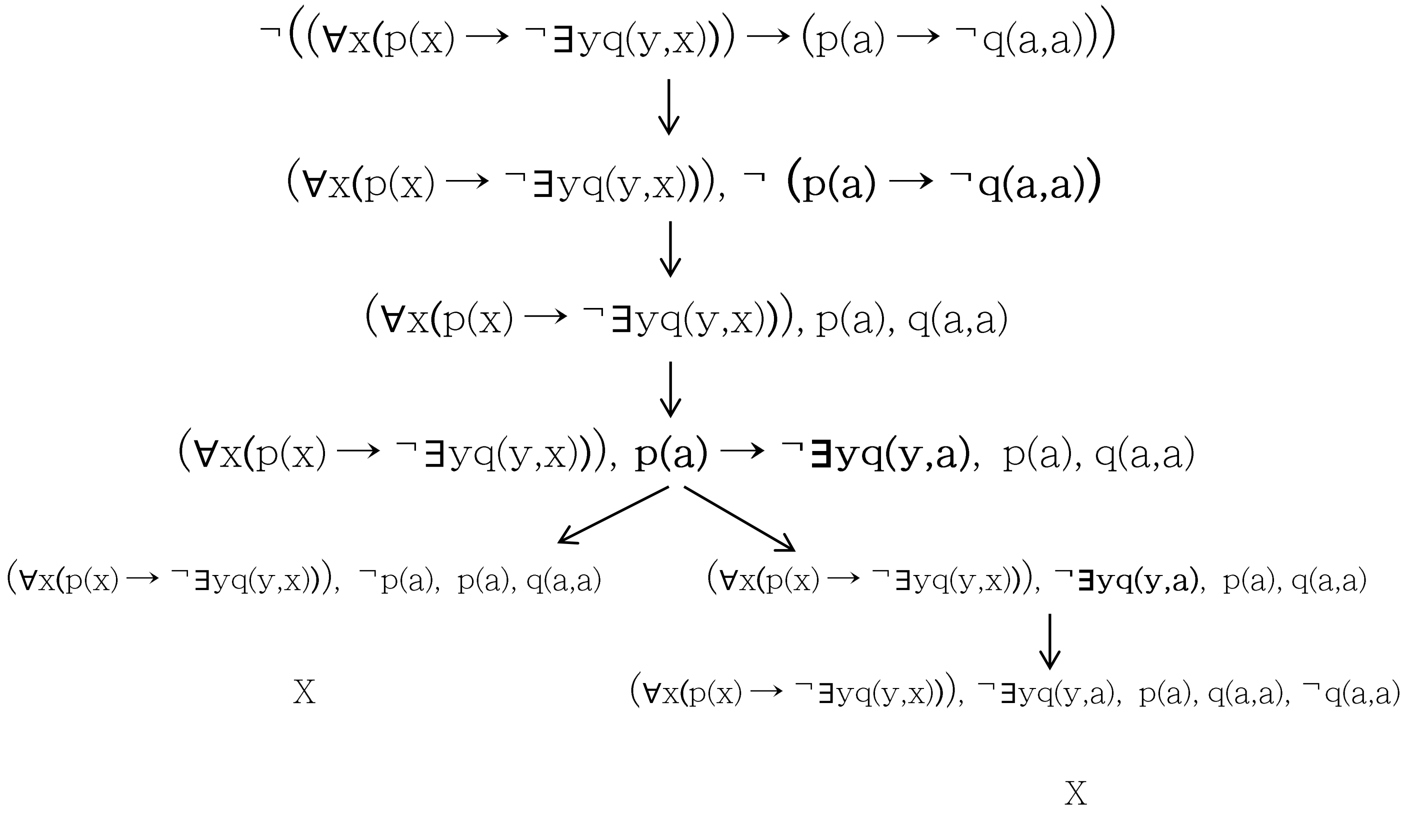
$$\forall x p(a,x),\ \neg(\exists x p(x,b)),\ \neg p(a,b),\ \neg p(b,b),\ (p(a,b)),\ p(a,a)$$

$$X$$

This shows that the formula $(\exists x \forall y p(x,y)) \longrightarrow (\forall y \exists x p(x,y))$ is valid.

Q 3.2

Tableau for $(\forall x(p(x) \rightarrow \neg \exists y q(y,x))) \rightarrow (p(a) \rightarrow \neg q(a,a))$:



This shows that the formula $(\forall x(p(x) \rightarrow \neg \exists y q(y,x))) \rightarrow (p(a) \rightarrow \neg q(a,a))$ is valid.